Universitatea "Alexandru Ioan Cuza" **Facultatea de Informatică**

Iaşi **Admitere, iulie 2013**

Barem de corectare la MATEMATICĂ

- Se acordă **10 puncte** din oficiu.
- Pentru orice soluţie corectă, chiar diferită de cea din barem, se acordă un punctaj corespunzător.

SUBIECTUL I **(30 de puncte)**

1.	$\dfrac{1}{1+a\cdot i}+\dfrac{1}{1-a\cdot i}=\dfrac{2}{1+a^2}, \forall a\in\mathbb{R}$	**2p**
	$\left\lvert\dfrac{2}{1+a^2}\right\rvert=\dfrac{2}{1+a^2}, \forall a\in\mathbb{R}$	**1p**
	$\dfrac{2}{1+a^2}\leqslant 2 \Leftrightarrow 1\leqslant 1+a^2, \forall a\in\mathbb{R}$	**2p**
2.	$\sum_{k=1}^{n} f(k)=\sum_{k=1}^{n} k^2=\dfrac{n(n+1)(2n+1)}{6}, \forall n\in\mathbb{N}^*$	**2p**
	$\sum_{k=1}^{2n} f(k)=\sum_{k=1}^{2n} k^2=\dfrac{2n(2n+1)(4n+1)}{6}, \forall n\in\mathbb{N}^*$	**1p**
	$\dfrac{2n(2n+1)(4n+1)}{6}\leqslant 8\dfrac{n(n+1)(2n+1)}{6} \Leftrightarrow 4n+1\leqslant 4n+4, \forall n\in\mathbb{N}^*$	**2p**
3.	$\sqrt[3]{3+\sqrt{x}}+\sqrt[3]{3-\sqrt{x}}=n\in\mathbb{N}\Rightarrow n^3=6+3n\sqrt[3]{9-x}$	**1p**
	$n^3=6+3n\sqrt[3]{9-x}\Rightarrow n^3<6+7n\Rightarrow n\in\{0,1,2\}$	**2p**
	$n=0\Rightarrow 0=6$ (fals), $n=1\Rightarrow x=\dfrac{368}{27}, n=2\Rightarrow x=\dfrac{235}{27}$	**2p**
4.	Numărul de cazuri posibile $n=49$	**2p**
	Numărul cazurilor favorabile $m=7$	**2p**
	Rezultatul: $p=\dfrac{m}{n}=\dfrac{1}{7}$	**1p**
5.	$mx+y+1=0$ şi $y=x^2\Rightarrow x^2+mx+1=0$	**1p**
	$\Delta=m^2-4\in\mathbb{N}\Rightarrow m^2-4=n^2$, pentru un $n\in\mathbb{N}$	**1p**
	$m=2p+1, n=2q+1\ (p,q\in\mathbb{N})\Rightarrow 4p^2+4p-4q^2-4q=4\Leftrightarrow (p-q)(p+q+1)=1$	**2p**
	$p-q$ şi $p+q$ au aceeaşi paritate $\Rightarrow (p-q)(p+q+1)$ par $\neq 1$	**1p**
6.	$\text{tg}\, x=t\Rightarrow \sin 2x=\dfrac{2t}{1+t^2}, \cos 2x=\dfrac{1-t^2}{1+t^2}$	**2p**
	$\sin 2x=\cos 2x\Rightarrow 2t=1-t^2\Rightarrow t_{1,2}=-1\pm\sqrt{2}$	**1p**
	$x<0\Rightarrow \text{tg}\, x\in\{-1\pm\sqrt{2}\}$	**2p**

SUBIECTUL al II-lea **(30 de puncte)**

1.a)	$A \cdot B = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix}, \forall A, B \in \mathcal{A}$	**2p**
	$A + B = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & a+b \\ 0 & 2 \end{bmatrix}, \forall A, B \in \mathcal{A}$	**2p**
	$A + B - I_2 = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix} = A \cdot B, \forall A, B \in \mathcal{A}$	**1p**
b)	$(\mathcal{A}, \cdot)$ este grup (demonstraţie) şi $(\mathbb{R}, +)$ este grup	**2p**
	$f : \mathcal{A} \to \mathbb{R}, f\left(\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\right) = a, \forall a \in \mathbb{R}$	**1p**
	f morfism de grupuri şi bijecţie	**2p**
c)	$X = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \Rightarrow X^n = \begin{bmatrix} 1 & nx \\ 0 & 1 \end{bmatrix}, \forall n \in \mathbb{N}^*$	**3p**
	$X^n = A \Leftrightarrow nx = a, n \in \mathbb{N}^*$	**1p**
	$x = \frac{a}{n} \Rightarrow X = \begin{bmatrix} 1 & a/n \\ 0 & 1 \end{bmatrix} \in \mathcal{A}, n \in \mathbb{N}^*$	**1p**
2.a)	$P = Q(X + \hat{2}) + m + \hat{3}$	**3p**
	câtul este $(X + \hat{2})$	**1p**
	restul este $(m + \hat{3})$	**1p**
b)	Q divide $P \Leftrightarrow m + \hat{3} = \hat{0}$	**3p**
	$m + \hat{3} = \hat{0} \Leftrightarrow m = \hat{2}$	**2p**
c)	$m = \hat{2} \Rightarrow P = (X^2 + \hat{1})(X + \hat{2})$	**1p**
	$(X^2 + \hat{1}) = (X^2 - \hat{4}) = (X - \hat{2})(X + \hat{2})$	**2p**
	$P = (X + \hat{3})(X + \hat{2})^2$	**2p**

SUBIECTUL al III-lea **(30 de puncte)**

1.a)	$f'(x) = 2xe^x + x^2e^x = (x^2+2x)e^x$	**1p**
	$f''(x) = (2x+2)e^x + (x^2+2x)e^x = (x^2+4x+2)e^x$	**1p**
	$f''(x) - 2f'(x) + f(x) = (x^2+4x+2)e^x - 2(x^2+2x)e^x + x^2e^x = 2e^x$	**2p**
	$e^x \geqslant 1, \forall x \geqslant 0 \Rightarrow f''(x) - 2f'(x) + f(x) \geqslant 2, \forall x \geqslant 0$	**1p**
b)	$\lim\limits_{x\to+\infty} \frac{f(x)}{x^5+1}$ nedeterminare $\left(= \frac{\infty}{\infty}\right)$	**1p**
	$\lim\limits_{x\to+\infty} \frac{x^2e^x}{x^5+1} \overset{\text{l'H.}}{=} \lim\limits_{x\to+\infty} \frac{(x^2+2x)e^x}{5x^4} \overset{\text{l'H.}}{=} \lim\limits_{x\to+\infty} \frac{(x^2+4x+2)e^x}{20x^3} \overset{\text{l'H.}}{=}$ $\lim\limits_{x\to+\infty} \frac{(x^2+6x+6)e^x}{60x^2} =$	**3p**
	$= \lim\limits_{x\to+\infty} \frac{1}{60}e^x = +\infty$	**1p**
c)	$f(0) = f'(0) = 0 \in \mathbb{N}$	**1p**
	$f^{(n)}(x) = \sum\limits_{k=0}^{n} C_n^k (x^2)^{(k)} (e^x)^{(n-k)} = \left[x^2 + 2nx + n(n-1)\right] e^x, \forall x \in \mathbb{R}, \forall n \geqslant 2$	**3p**
	$f^{(n)}(0) = n(n-1) \in \mathbb{N}, \forall n \in \mathbb{N}, n \geqslant 2$	**1p**
2.a)	$\int f_2(x)\,dx = \int \cos^2 x\,dx = \int \frac{\cos 2x + 1}{2}\,dx = \frac{1}{4}\sin 2x + \frac{x}{2} + C$, cu un $C \in \mathbb{R}$	**5p**
b)	$I = \int_0^{\frac{\pi}{2}} \cos^4 x = \int_0^{\frac{\pi}{2}} \cos^3 x\,(\sin x)'\,dx = \cos^3 x \sin x \Big\vert_0^{\frac{\pi}{2}} + 3\int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x\,dx =$ $3\int_0^{\frac{\pi}{2}} (\cos^2 x - \cos^4 x)\,dx =$	**2p**
	$= -3I + 3\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = -3I + \frac{3}{4}\sin 2x \Big\vert_0^{\frac{\pi}{2}} + \frac{3x}{2}\Big\vert_0^{\frac{\pi}{2}} = -3I + \frac{3\pi}{4}$	**2p**
	$4I = \frac{3\pi}{4} \Rightarrow I = \frac{3\pi}{16}$	**1p**
c)	$I_n = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \int_0^{\frac{\pi}{2}} (\sin x)' \cos^{n-1} x\,dx =$	
	$= \sin x \cos^{n-1} x \Big\vert_0^{\frac{\pi}{2}} + (n-1)\int_0^{\frac{\pi}{2}} \sin^2 x \cos^{n-2} x\,dx =$	**2p**
	$= (n-1)\int_0^{\frac{\pi}{2}} (1 - \cos^2 x)\cos^{n-2} x\,dx = (n-1)I_{n-2} - (n-1)I_n \Rightarrow$	**2p**
	$\Rightarrow nI_n = (n-1)I_{n-2}, \forall n \in \mathbb{N}, n \geqslant 2$	**1p**